Sensitivity analysis of the balance stability region in legged mechanisms

Carlos Gonzalez, Carlotta Mummolo, Joo H. Kim[1]

Department of Mechanical and Aerospace Engineering

New York University, Brooklyn, NY, USA

[1]joo.h.kim@nyu.edu

Summary

In order to determine how close a legged system is to actual falling, the simultaneous influence of multiple factors must be taken into account, such as system parameters (e.g., actuation and angle limits, foot size, etc.), physics initial and final conditions , and center of pressure limits. These factors are responsible for setting a boundary to all the possible balanced state trajectories that can be generated for a given legged system. In this work, we address the question of how sensitive this balance-stability region is to variations in its different system parameters, e.g., to perturbed torque limits and to perturbed foot size limits. We use dynamics to obtain numerical solutions, and optimization tools (sensitivity analysis of the optimal solution, which are in progress) to obtain analytical solutions to this question.

Introduction

The mechanical design of legged systems is not a trivial task due to the high complexity and interplay of the parameters to be considered, especially when considering a design purpose, e.g., energy-efficient vs. highly stable (Collins & Ruina 2005). The question of stability of the system often comes at hand in the design process, especially when making changes in the mechanical design, which is often dependent not only on simulations but also on heuristics. Common parameters disputed in the design process are the selection of the motors, often based on their weight and torque and joint speed limitations and the dimensions and the weight of the foot. The work presented here is focused on the development of theoretical constructs that make use of tools from dynamics and optimization theory to provide incentive of the different impact and interplay of some of these design parameters.

Based on a novel computational framework for the balance stability regions of legged system in the center of mass (COM) state space (Mummolo et al. 2017), this work proposes a sensitivity-based method to predict the effects that variations on system parameters have on the dimensions of the balance stability boundary. In particular the biped parameters considered are foot size, also called foot support region (FSR) and torque limits, which have direct impact on the limits of center of pressure displacement. Numerical results shows that the capability to attain a balanced condition (e.g., avoiding to fall) is enhanced by increasing either the foot support base or the torque capacity at the joint (i.e., by having a stronger joint).

Methods

A simple model used to characterize a bipedal system is the inverted pendulum model, described by:

$$\ddot{\theta} = f(\theta, \tau) \tag{1}$$

with joint angle acceleration $\ddot{\theta}$ being a nonlinear function of joint angle θ and joint torque τ. This model is considered jointly with the biped constraints:

$$F_y \geq 0 \tag{2}$$

$$|F_x| \leq \mu F_y \tag{3}$$

$$\mathrm{FSR}_l \leq x_{\mathrm{COP}} \leq \mathrm{FSR}_r \tag{4}$$

where F_x and F_y are the horizontal and vertical ground reaction forces, respectively, μ is the coefficient of static friction, x_{COP} is the location of the Center of Pressure (COP), and FSR_l and FSR_r are the left and right ends of the FSR, respectively.

The above constrained system, with its specified parameters, is implemented in an optimization formulation to find the minimum and maximum value of $\dot{\theta}$ at a certain time instant t_0, such that the following constraints are satisfied: constraints (1) – (4) for $[t_0, T]$, specified position θ at t_0, final condition of static balance at $t_0 + T$, and the system's joint angle, speed, and torque limitation for $[t_0, T]$. Computing these velocity extrema for each discretized value of θ, the points in the balance stability region can be evaluated (Mummolo et al. 2017). The resulting region for a specified legged system identifies the set of states from which a final balanced state can be attained given proper, unspecified controller.

In the work presented here, these results are extended to study the effects of: (*i*) joint torque limits variation on the stability region, and (*ii*) of FSR variation on the stability region. Through a parametrical study, these effects are studied numerically first, by iteratively computing the boundaries of a system of interest with nominal system parameters m, L, τ^{LB}, τ^{UB}, FSR_l, and FSR_r, and then computing the boundary for a system with one modified parameters at a time (e.g., m, L, $\tilde{\tau}^{\mathrm{LB}}$, $\tilde{\tau}^{\mathrm{UB}}$, FSR_l, and FSR_r). In addition, a sensitivity-based method to predict the results of the parametrical study is proposed. Since the points of the balance stability boundary is a direct result of a constrained optimization problem, the effects that variations on system parameters have on the dimensions of the balance stability boundary could be predicted using nonlinear constrained optimization theory. A sensitivity analysis can be performed on the optimization problem solved at each point of the balance stability boundary, where the cost function being minimized is:

$$f_0(x) = \dot{x}(t_0) \tag{5}$$

where $\dot{x}(t_0)$ denotes the initial velocity of the COM in the *x*-direction. For instance, in the case of the sensitivity of the boundary with respect to the FSR variation, constraint (4) is written in standard form with:

$$g_1 = x_{\mathrm{COP}} - \mathrm{FSR}_r \le 0 \tag{6}$$

$$g_2 = \mathrm{FSR}_l - x_{\mathrm{COP}} \le 0 \tag{7}$$

where the sensitivity equation is given by:

$$\delta f_0^* = -u_1^* e_1 - u_2^* e_2 \tag{8}$$

with u_1^* and u_2^* being the Lagrange multipliers (of the optimal solution) associated to g_1 and g_2, respectively, while e_1 and e_2 being the variation of the RHS of g_1 and g_2, respectively. Hence, the perturbed balance stability regions for perturbed FSR's can be directly computed through (8).

Results

For a nominal system, modeled as a nonlinear inverted pendulum subject to the biped constraints (2) - (4), the parametrical study on the stability boundary region is performed first by varying torque limits at the ankle joint (Figure 1) and then by varying the FSR dimensions (Figure 2), while the rest of the parameters are held constant. It can be seen that as the torque limit decreases, the stability boundary of the system shrinks. Similarly, as the torque limits increase, the stability boundary expands.

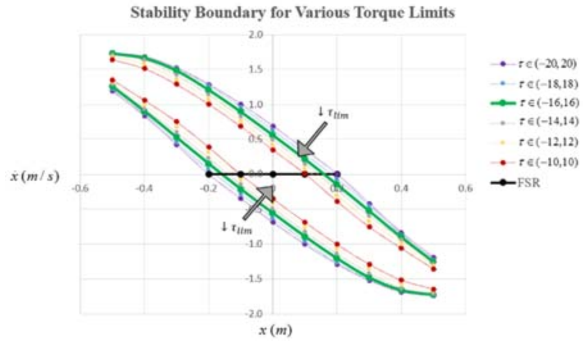

Figure 1. Balance stability boundary with perturbed torque limits.

Slight changes in the FSR have a greater impact on the balance stability region, as compared to the changes seen in Figure 1.

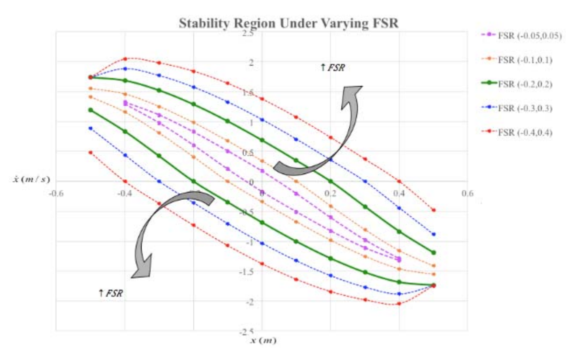

Figure 2. Balance stability boundary with variations on FSR's.

Discussion

This work makes use of a recent boundary stability framework for legged systems to investigate the sensitivity of stability boundary regions in a perturbed system. The results, either from parametrical study of predicted from sensitivity analysis, could provide insights in the model-based design of stable legged mechanisms.

References

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